COMPARISON OF THE FORM FACTORS OF TIN ISOTOPES

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Abstract

The purpose of this research is to obtain the form factors of ^{116}Sn, ^{118}Sn and ^{124}Sn nuclei using two parameter Fermi model (2PF) . The structural parameters, namely radius parameter (c) and the skin thickness parameter (z) of ^{116}Sn, ^{118}Sn and ^{124}Sn taken from the experimental data are used to get the charge density distribution. After getting the charge density distribution, the root mean square radius and the form factors are calculated.

Key words: Charge density distribution, form factor, root mean square radius.

Introduction

There are two types of nuclear distributions which are nuclear charge distribution and nuclear matter distribution. We are dealing with nuclear charge density distribution. In theory, nuclear charge density distributions are expressed in various form factors depending upon nuclear model. In the present work nuclear distribution on charge distribution are given in two parameter Fermi model (2PF) and three parameter Fermi model (3PF) respectively. The parameters consist in these models are obtained from scattering experiments. A study of electron scattering would be a method of measuring the distribution of the static electromagnetic field of the nucleus. The scattering of β-particles are not monoenergetic, they have low momentum and suffer, in addition to large single scatters, severe multiple scattering effects. This property prevented clear results being obtained from early measurements of electron scattering. However, the scattering of more energetic electrons (>100 MeV) is a very important tool in the investigation of the nuclear size.

In atomic physics, the boundary of an atom is not sharp since the wave function of the outer electrons decreases monotonically: therefore we should expect a similar situation to occur in nuclear physics. As we have seen, when Rutherford was investigating the scattering of α-particle by gold, his scattering

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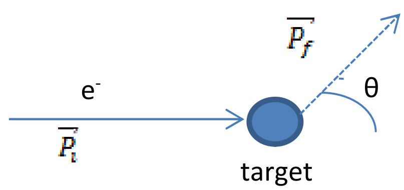
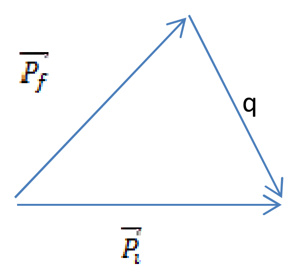
measurements were in agreements with the predictions found assuming that the interaction was the Coulomb potential between two point charges. Thus for gold at the classical distance of closest approach that could be reached with the α-particle energies that Rutherford had available, he was unable to detect any deviations which might indicate that the charges on α-particle and the gold nucleus were point like. However, using light nuclei as target, Rutherford found deviations which indicated a breakdown in the Coulomb's law when the classical distance of closest approach was about 10^{-4} m. These deviations occur at short distances of approach, because the finite charge density distributions overlap and the strong nuclear forces which exist between the α-particle and nucleus come into play. It is necessary to distinguish between the distribution of the source of electric fields (proton) and the source of nuclear fields (protons and neutrons), although these sources are probably strongly linked. To make progress we should use a probe for one which does not feel the other. The electron is coupled to an electric field due to charge sources but does not experience the nuclear force. Conversely, the neutron is electrically neutral but does not experience the nuclear force. Thus separate scattering experiments with these two particles should, in principle, go some way to measuring these two distributions. So, electron scattering experiments are tools to obtain charge density distribution of nuclei while neutron scattering experiments are used to get matter distribution.

Relation between Cross Section and Form Factor

Form Factors are an intuitive and simple tool used to describe the scattering particles from extended targets. Here we will show how the Form Factor comes about in the context of the scattering of spinless electrons. A discussion of the more rigorous description, which includes electron spin a magentic moment, will follow. The typical set up has an electron beam with initial momentum $\vec{P}_i$ directed at the scattering target.





The electrons are deflected through an angle θ with a final momentum $\vec{P}_f$.We define the momentum transfer as the vector $\vec{q} = \vec{P}_i - \vec{P}_f$. As with many scattering experiments, the quantity we are interested in is the differential cross section $\frac{d\sigma}{d\Omega}$ of our scattered electrons off our target. This quantity can be measured in the lab and easily connected to QM scattering theory in order to confirm theory and provide insight to the physical processes at play. Earlier we had shown that the differential cross section is related to the scattering amplitudes through the relation:

$$\frac{d\sigma}{d\Omega} = \frac{k}{k_i} |f(\theta, \emptyset)|^2 \quad (1)$$

The scattering amplitudes $f(\theta, \emptyset)$ can be obtained in approximate form using the Born Approximation. To first order (and up to a normalization) the Born Approximation can be written as:

$$f_{B1=} \left\langle \emptyset_{\vec{k}_f} \middle| V \middle| \emptyset_{\vec{k}_i} \right\rangle \quad (2)$$

$$\int \emptyset^*_{\vec{k}_f}(\vec{r})\, V(\vec{r}) \emptyset_{\vec{k}_i}(\vec{r}) d^3(\vec{r}) \quad (3)$$

In the first Born Approximation the initial incoming wave and the outgoing waves are assumed to be plane waves of the form:

$$\emptyset_{\vec{k}_i}(\vec{r}) = e^{i\vec{k}_i \cdot \vec{r}}$$

$$\emptyset_{\vec{k}_f}(\vec{r}) = e^{i\vec{k}_f \cdot \vec{r}}$$

We will also define the momentum transfer as $\frac{\vec{q}}{\hbar} = \vec{k}_i - \vec{k}_f$. Making use of these definitions the first Born Approximation can be written as:

$$f_{B1=} \int e^{\frac{i\vec{q}\cdot\vec{r}}{\hbar}} V(\vec{r}) d^3\vec{r} \quad (4)$$

This result is still quite general; in order to proceed we will need to assume a specific form for the potential, V($\vec{r}$). We can describe an extended charge distribution by Ze $\rho(\vec{r})$ with

$$\int \rho(\vec{r}) d^3\vec{r} = 1 \quad (5)$$

In this case, the potential experienced by an electron located at $(\vec{r})$ is given by the Coulomb potential:

$$V(\vec{r}) = \frac{-ze^2}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r})}{\left|\vec{r}-\vec{r'}\right|}\, d^3\vec{r'} \qquad (6)$$

Substitute this potential into the general expression for the first Born Approximation to the scattering amplitudes $f\ (\theta\ ,\emptyset)$ in (eq. 4)

$$f_{B1=} \frac{-ze^2}{4\pi\varepsilon_0} \int e^{\frac{i\vec{q}\cdot\vec{r}}{\hbar}} \frac{\rho\left(\vec{r'}\right)}{\left|\vec{r}-\vec{r'}\right|}\, d^3\vec{r'}\, d^3\vec{r} \qquad (7)$$

Make the substitution $\vec{R} = \vec{r} - \vec{r'}$ and noting that $d^3\vec{R} = d^3\vec{r}$,

$$f_{B1=} \frac{-ze^2}{4\pi\varepsilon_0} \int e^{\frac{i\vec{q}\cdot\vec{r}}{\hbar}} \frac{\rho\left(\vec{r'}\right)}{|\vec{R}|}\, d^3\vec{r'}\, d^3\vec{R} \qquad (8)$$

$$\frac{-ze^2}{4\pi\varepsilon_0} \int e^{\frac{i\vec{q}\cdot\vec{r}}{\hbar}} \frac{\rho\left(\vec{r'}\right)}{|\vec{R}|}\, d^3\vec{r'}\, d^3\vec{R} \qquad (9)$$

$$\frac{-ze^2}{4\pi\varepsilon_0} \int \frac{e^{\frac{i\vec{q}\cdot\vec{R}}{\hbar}}}{|\vec{R}|}\, d^3\vec{R} \left[\int e^{\frac{i\vec{q}\cdot\vec{r'}}{\hbar}} \rho\left(\vec{r'}\right) d^3\vec{r'}\right] \qquad (10)$$

This bracket factor is known as the 'Form Factor', F(q).

$$\mathrm{F(q)} = \int e^{\frac{i\vec{q}\cdot\vec{r'}}{\hbar}} \rho\left(\vec{r'}\right) d^3\vec{r'} \qquad (11)$$

It can be shown that when the expression for f_{B1} is used to determine $\frac{d\sigma}{d\Omega}$,

$$\frac{d\sigma}{d\Omega} = \frac{k}{k_i} f_{B_1}^2 |F(q)|^2 \qquad (12)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{point} = \frac{k}{k_i} f_{B_1}^2 \qquad (13)$$

So, Relation between cross section and form factor is

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} |F(q)|^2 \qquad (14)$$

To provide some insight into the meaning of form factors and probability distribution, we shall connect F (q^2) to nuclear radius and give examples of the relation between form factor and probability distribution. For qR<<1, where R is approximately the nuclear radius, the exponential in equation (11) can be expanded, and F(q^2) becomes

$$F(q^2) = 1- \frac{1}{6\hbar^2}\, q^2 \langle r^2 \rangle + \cdots \qquad (15)$$

where $\langle r^2 \rangle$ is defined by

$$\langle r^2 \rangle = \int d^3r\, r^2 \rho(r) \qquad (16)$$

and is called the mean-square radius.

Two Parameter Fermi Model

In our calculation, two parameter Fermi model is used to calculate the root mean square radius and charge density distribution.

For two parameter Fermi model, charge density distribution is

$$\rho(r) = \frac{\rho_0}{\left(1+ e^{(r-c)/z}\right)} \qquad (17)$$

Where, c = the radius parameter

z = the skin thickness parameter

The variation of form factor with momentum transfer is

$$F(q) = \frac{4\pi}{qZe} \int \rho(r) r sin(qr) dr \qquad (18)$$

The parameters of two parameter Fermi model are expressed in Table.

No.	Nucleus	c (fm)	z (fm)
1	^{116}Sn	5.358	0.550
2	^{118}Sn	5.412	0.560
3	^{124}Sn	5.490	0.534

Calculation of Charge Density Distribution, Form Factor and RMS for Two Parameter Fermi Model

Density distribution for two parameter Fermi model is

$$\rho(r) = \frac{\rho_0}{\left(1+ e^{(r-c)/z}\right)} \tag{19}$$

Where, c = the radius parameter

z = the skin thickness parameter

The normalization condition is

$$\int \rho(r) 4\pi r^2 dr = Ze \tag{20}$$

$$\int \rho(r) r^2 dr = \frac{Ze}{4\pi} \tag{21}$$

By substituting equation (19) in equation (21),

Let, $$\text{TER}=\int \frac{1}{\left(1+e^{(r-c)/z}\right)} r^2 dr \tag{22}$$

$$\rho_0 = \frac{1}{TERM} \times \frac{Ze}{4\pi} \tag{23}$$

By substituting equation (23) in equation (19),

$$\rho\,(r) = \frac{1}{(TERM)^2}\left(\frac{Ze}{4\pi}\right) \tag{24}$$

The variation of form factor with momentum transfer is

$$\text{F(q)}=\frac{4\pi}{qZe}\int \frac{\rho_0}{\left(1+exp((r-c)/z)\right)} r sin(qr) dr \tag{25}$$

$$\text{F(q)}=\frac{1}{q}\,\frac{1}{TERM}\int \frac{1}{\left(1+exp((r-c)/z)\right)} r sin(qr) dr \tag{26}$$

Root mean square radius is

$$\langle \text{r}^2\rangle^{1/2} = \frac{4\pi}{Ze}\int \rho(r) r^4 dr$$

$$\langle \text{r}^2\rangle^{1/2} = \frac{4\pi}{Ze}\int \frac{1}{(TERM)^2}\,\frac{Ze}{4\pi} r^4 dr$$

$$\langle \text{r}^2\rangle^{1/2} = \int \frac{r^4}{(TERM)^2}\, dr \tag{27}$$

The charge density distributions, the form factors and root mean square radii are solved numerically by using a FORTRAN code.

Results and Discussion

The charge density distribution, form factor and the root mean square radius of ^{116}Sn, ^{118}Sn and ^{124}Sn are calculated by using two perimeter Fermi model (2PF). The parameters we used in our calculation for Sn isotopes are shown in Table (1).

Table 1. The parameters of two parameter Fermi model

No.	Nucleus	c (fm)	z (fm)
1	^{116}Sn	5.358	0.550
2	^{118}Sn	5.412	0.560
3	^{124}Sn	5.490	0.534

By using these parameters, charge density distributions are calculated and the resultant distributions are displayed in figure (1), (2) and (3). From these distributions, we calculated the form factors and root mean square radii for the Sn isotopes. The calculated form factors are displayed in figure (4), (5) and (6). The root mean square radii are displayed in Table (2).

Table 2. Comparison for RMS value for ^{116}Sn, ^{118}Sn and ^{124}Sn

No.	^{116}Sn	^{118}Sn	^{124}Sn
RMS radii	4.530609	4.576880	4.596027
Experimental Value	4.53	4.58	4.60

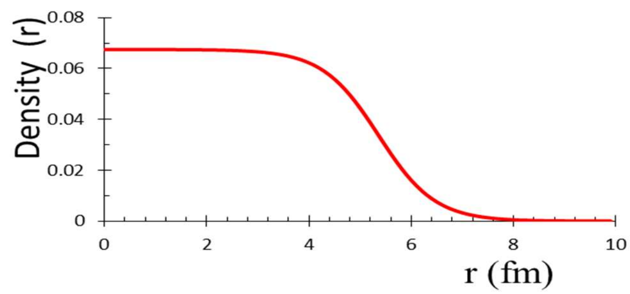


Figure 1. Charge density distribution of ^{116}Sn

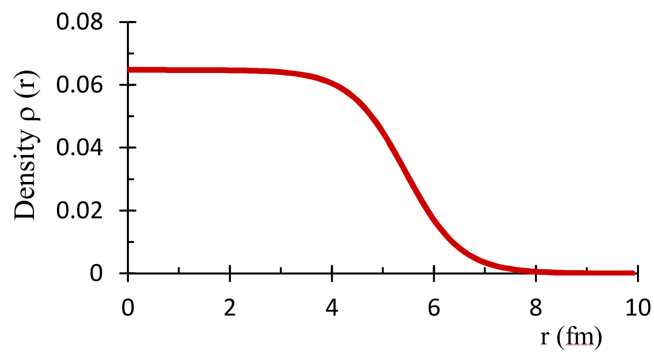


Figure 2. Charge density distribution of ^{118}Sn

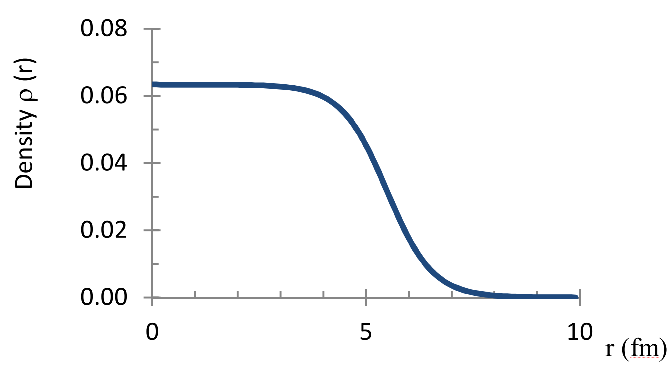
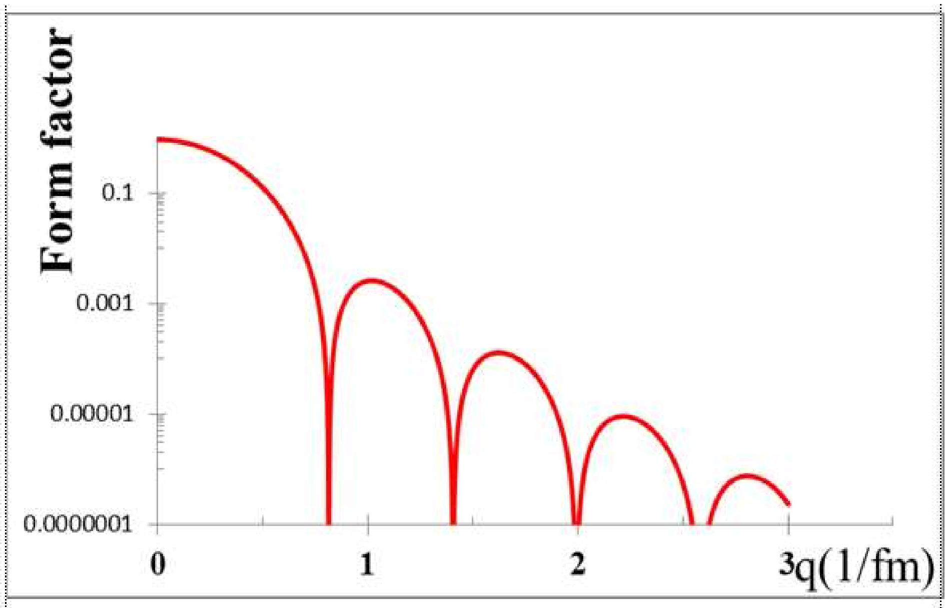


Figure 3. Charge density distribution of ^{124}Sn

Variation of Form factor with momentum transfer for ^{116}Sn



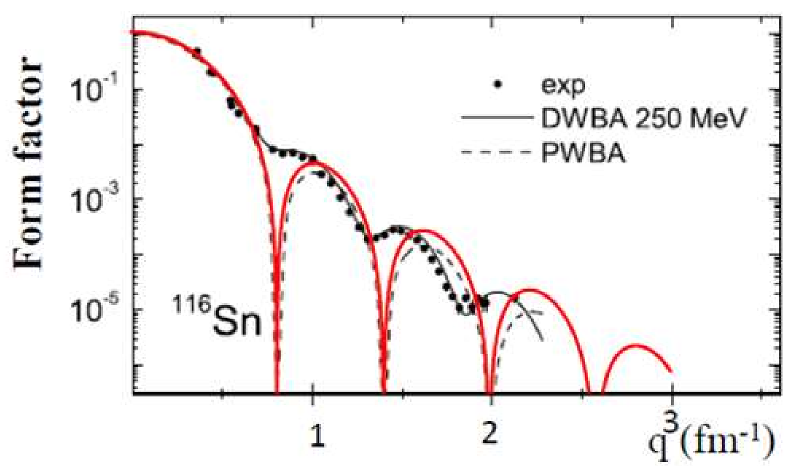
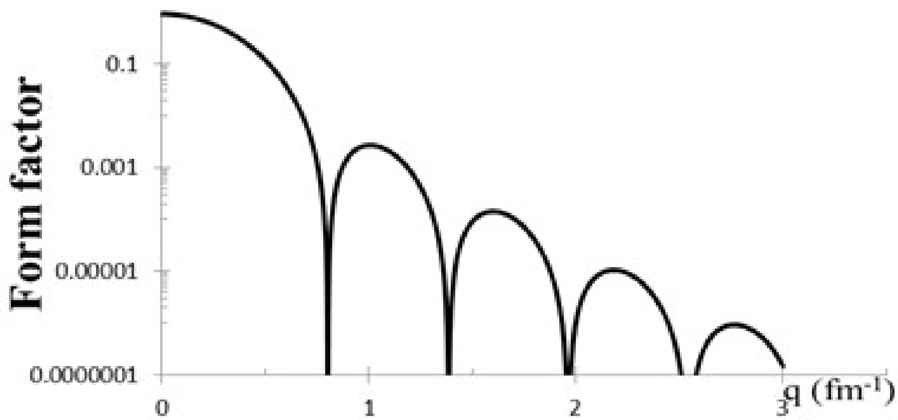


Figure 4. Comparison of experimental and calculated results of form factor for ^{116}Sn

Variation of Form factor with momentum transfer for ^{118}Sn



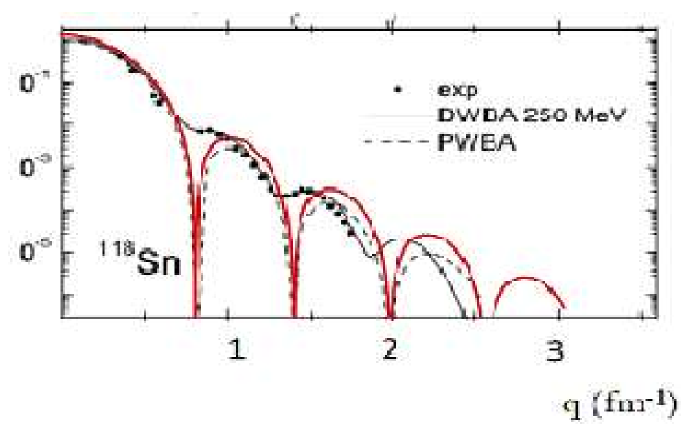
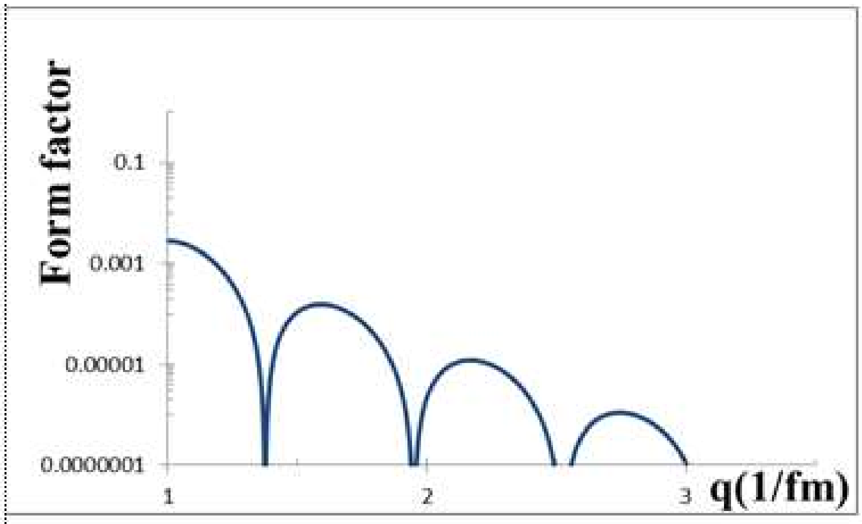


Figure 5. Comparison of experimental and calculated results of form factor for ^{118}Sn

<u>Variation of Form factor with momentum transfer for ^{124}Sn</u>



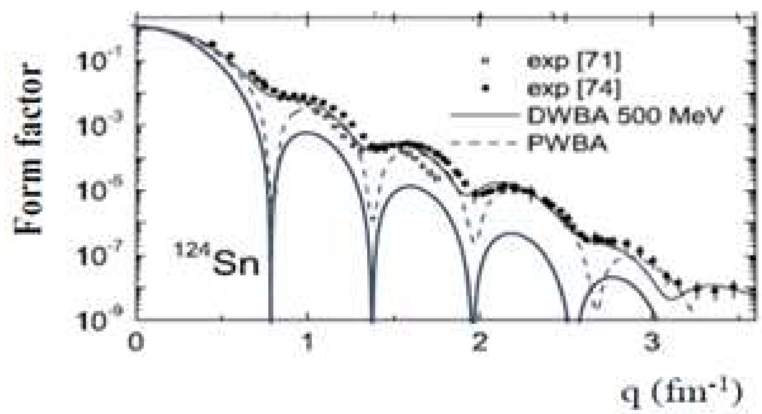


Figure 6. Comparison of experimental and calculated results of form factor for ^{124}Sn

Conclusion

In our calculation, we have calculated the charge density distributions in two parameter Fermi model for Sn isotopes. From these density distributions, we calculated the form factors and root mean square radii of Sn isotopes. In our results, we concluded that charge form factors and rms values which are calculated using the charge density of two parameter Fermi model is good agreement with the experimental value.

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